DS 574 Algorithmic Mechanism Design	Lecture #18 Worksheet
Fall 2023	Prof. Kira Goldner

Behavioral Economics I

Thus far, we have assumed that an agent's utility is *quasilinear* and that an agent maximizes *expected utility*:

$$u_i(b_i, \mathbf{b}_{-i}) =$$

However, humans do not always maximize this objective. Some environments and some individuals lead to different behavior. Consider the following.

Gamble A	\$10 w.p. 1
Gamble B	$\begin{cases} Y = \$20 & \text{w.p. } 0.5 \\ \$0 & \text{w.p. } 0.5 \end{cases}$

Which gamble do you prefer? For what value of Y would you prefer gamble B?

What if the gambles instead look like the following:

Gamble A	\$1000 w.p. 1
Gamble B	$\begin{cases} Y + \$990 & \text{w.p. } 0.5 \\ \$990 & \text{w.p. } 0.5 \end{cases}$

How can we explain this behavior?

The Expected Utility Theory of von Neumann Morgenstern [8] says that this can be explained by agents having:

One can arrive at the theory that agents are *expected utility* maximizers from a few small number of natural axioms:

- Completeness: For any two alternatives (including lotteries) A and B, an agent can rank these. (Either $A \succeq B$ or $B \succeq A$ or both.)

- Transitivity: If $A \succeq B$ and $B \succeq C$ then $A \succeq C$.
- Continuity: Let $A \succeq B \succeq C$. Then there exists some $p \in [0,1]$ such that B is equally preferred to $pA + (1-p)C$.
- Independence of Irrelevant Alternatives: If $A \succeq B$, then $tA + (1-t)C \succeq tB + (1-t)C$ for every lottery C and real $t \in [0,1]$.

If these axioms are satisfied, then the agent is rational, and their preferences can be represented by maximizing expected utility with respect to a utility function.

So what should $m(\cdot)$ look like in the above formulation of utility? EUT says that if $m(\cdot)$ is concave, that is, agents have *decreasing marginal returns* for money, then this explains risk-averse behavior. (And convex functions explain risk-seeking behavior.)

The Calibration Theorem (Rabin [9]):

Gamble A	$\begin{cases} \$11 & \text{w.p. } 0.5 \\ -\$10 & \text{w.p. } 0.5 \end{cases}$
Gamble B	$\begin{cases} \$Y & \text{w.p. } 0.5 \\ -\$100 & \text{w.p. } 0.5 \end{cases}$

Suppose an agent rejects gamble A, deciding it has negative utility for them. Under EUT, for what values of $\$Y$ will the agent also reject gamble B?

Here's another experiment that was actually run, called the Allais Paradox [1]. Which gamble would you prefer?

Gamble A	\$1M w.p. 1
Gamble B	$\begin{cases} \$1\text{M} & \text{w.p. } 0.89 \\ \$5\text{M} & \text{w.p. } 0.10 \\ \$0 & \text{w.p. } 0.01 \end{cases}$

EUT cannot explain this. This motivated *cumulative prospect theory* (CPT) [5, 10]. CPT accounts for three things not seen in EUT:

- Probability weighting.
- Diminishing sensitivity.
- Loss aversion.

Probability weighting:

$$u_i(b_i, \mathbf{b}_{-i}) = y_i(x_i(b_i, \mathbf{b}_{-i})) \cdot m(v_i - p_i(b_i, \mathbf{b}_{-i})).$$

Suppose $y(\cdot)$ is convex, like x^2. Then you value certain probabilities (0 and 1) the same, but you underweight uncertain probabilities.

Diminishing sensitivity is with respect to the reference point (see next). The closer to the reference point, the more sensitive.

Loss Aversion

Loss Aversion if the concept that losses weigh more heavily on an agent than gains. This requires evaluating gains and losses relative to some reference point.

An *alternative* (think: action) is a one-dimensional random variable c. Given some increasing utility function $m : \mathbb{R} \to \mathbb{R}$, faced with choice set C, an EU maximizer picks $c \in C$ to max $\mathbb{E}[m(c)]$.

Incorporating loss aversion: Given a reference point $r \in \mathbb{R}$ and parameters $\eta \geq 0, \lambda \geq 1$:

$$n(c \mid r) = \begin{cases} \eta(m(c) - m(r)) & \text{if } m(c) - m(r) > 0 \\ \eta\lambda(m(c) - m(r)) & \text{otherwise} \end{cases}$$

Faced with choice set C, a loss-averse decision maker picks $c \in C$ to max $\mathbb{E}[m(c) + n(c \mid r)]$. Then EUT is a special case. Think of $m(\cdot)$ as 'material' utility—m is the classical part, and n is the gain/loss part. This is a good compromise between accuracy and tractability.

Let's use this to explain the difference in small/large stakes risk aversion. Suppose $m(c) = c$, $\eta = 1$, $\lambda = 1.5$. Let the reference point r refer to status quo wealth w. Then

$$m(w + x) + (w + x \mid r) = w + x + \begin{cases} x & \text{if } x > 0 \\ 1.5 & \text{otherwise} \end{cases}$$

The decision maker will accept the coin flip if and only if the ratio of gain to loss as at least 1.25.

The Endowment Effect (Kahneman Knetsch Thaler [4]): In a famous experiment, coffee mugs were randomly allocated to half of the class. The following (not truthful but can be made so) market-clearing mechanism was used. The students were asked for a list of prices: "At a price of $\$p$ will you sell/buy?" to which they answered yes or no. Then this was used to find a market-clearing price.

If we randomly allocate 22 mugs to 44 students, we should expect:

What we see:

One explanation given is that people have transaction costs: it's a hassle to trade. However, if you use vouchers instead of mugs, you see exactly 11 trades.

It turns out that loss aversion with multiple dimensions can explain this. You have a reference point for the mug (whether you own it or not) and a reference point for money (how much you start with). You would model this as maximizing

$$\sum_i [m_k(c_k) + n_k(c_k \mid r_k)]$$

for a variety of reference points r_k.

The Winners' Curse and Cursed Equilibrium

Recall the *wallet game* with interdependent valuations. Each agent i knows the amount of money in their wallet, s_i, and their value is the amount of money in total in the room, $v = \sum_i s_i$. Whoever wins the auction (has the highest bid) for the sum of wallets gets all of the cash from the wallets. Let's suppose we use an English (ascending) auction. For simplicity, suppose $s_i \sim U[0, 1]$. How should the players bid if they only know their own signal? Let's suppose there are only two agents: you and your opponent.

What would you bid?

Naive strategy:

BNE strategy:

Crux:

Cursed equilibrium (Eyster and Rabin [3]) asserts: Players correctly understand the marginal distributions $\mathbf{b}_{-i} \mid s_i$ and $\mathbf{s}_{-i} \mid s_i$, but under appreciate how $\mathbf{b}_{-i}$ and $\mathbf{s}_{-i}$ are related.

It's as though with some probability χ, independent of $\mathbf{s}_{-i}$, all opponents play the unconditional distribution of $\mathbf{b}_{-i}$.

Let $\bar{\sigma}$ denote the strategy of your opponents averaged over their types. That is,

$$\bar{\sigma}_{-i}(\mathbf{b}_{-i} \mid s_i) = \sum_{\mathbf{s}_{-i}} \Pr[\mathbf{s}_{-i} \mid s_i] \cdot \sigma_{-i}(\mathbf{b}_{-i} \mid \mathbf{s}_{-i}).$$

Then σ is a χ-cursed equilibrium if for all i, θ_i: for all b_i such that $\sigma_i(b_i \mid s_i) > 0$:

$$b_i \in \arg\max_{b_i'} \sum_{\mathbf{s}_{-i}} \Pr[\mathbf{s}_{-i} \mid s_i] \sum_{\mathbf{b}_{-i}} u_i(b_i', \mathbf{b}_{-i}, s_i, \mathbf{s}_{-i}) \cdot \left[\chi\bar{\sigma}_{-i}(\mathbf{b}_{-i} \mid s_i) + (1-\chi)\sigma_{-i}(\mathbf{b}_{-i} \mid s_i)\right]$$

If $\chi = 0$, this is just a BNE. If $\chi = 1$, this is fully cursed—you operate as if everyone else is playing an average over all types and ignoring their true type.

Another example: Seller has value equal to their private signal s_S. Buyer has value $1.5s_S$. The seller will accept a price p if and only if $p \geq s_S$. What price p should the buyer offer? Say $s_S \sim U[0, 1]$.

The BNE has:

Other concepts: Quantal response [6, 7]: People choose actions randomly, but choose higher-utility outcomes with high probability.

Level-k Thinking [2]: We assume that each agent performs a finite number of steps of strategic reasoning.

- Level-0: Some default nonstrategic distribution of play (perhaps uniform).
- Level-1: Best respond to level-0 players.
- $\cdots$
- Level-k: Best respond to level $k-1$, or possibly levels $\{0, 1, \ldots, k-1\}$.

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References

[1] Maurice Allais. Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine. *Econometrica: journal of the Econometric Society*, pages 503–546, 1953.

[2] Vincent P Crawford and SESS Distinguished Lecture. Outguessing and deception in novel strategic situations. *School of Economics and Social Sciences Distinguished Lecture, Singapore Management University*, 2004.

[3] Erik Eyster and Matthew Rabin. Cursed equilibrium. *Econometrica*, 73(5):1623–1672, 2005.

[4] Daniel Kahneman, Jack L Knetsch, and Richard H Thaler. Experimental tests of the endowment effect and the coase theorem. *Journal of political Economy*, 98(6):1325–1348, 1990.

[5] Daniel Kahneman and Amos Tversky. Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2):263–292, 1979.

[6] Richard D McKelvey and Thomas R Palfrey. Quantal response equilibria for normal form games. *Games and economic behavior*, 10(1):6–38, 1995.

[7] Richard D McKelvey and Thomas R Palfrey. Quantal response equilibria for extensive form games. *Experimental economics*, 1:9–41, 1998.

[8] J von Neumann, Oskar Morgenstern, et al. Theory of games and economic behavior, 1947.

[9] Matthew Rabin. Diminishing marginal utility of wealth cannot explain risk aversion. 2000.

[10] Amos Tversky and Daniel Kahneman. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, 5:297–323, 1992.