DS 574 Algorithmic Mechanism Design	**Lecture #7 Worksheet**
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Ascending Auctions

In *ascending auctions*, an auctioneer initializes prices for each item, iteratively raises the prices, and bidders decide which items to bid on in each round. Sometimes *activity rules* are enforced, e.g., once you drop out on an item, you can not bid on it again.

The most famous ascending auction is the single-item version, the English Auction.

The English Auction(ε):

1. Initialize the item's price p_0 to
2. The initial set S_0 of "active bidders" (willing to pay p_0 for the item) is
3. For iteration $t = 1, 2, \ldots,$:
   (a) Ask the set of active bidders S_{t-1}:

   $S_t =$

   (b) If $|S_t| \leq 1$:

   (c) Otherwise, p_t

Benefits of using ascending auctions:

- Ascending auctions are easier for bidders.
- Less information leakage.
- Transparency.
- Potentially more seller revenue.
- When there are multiple items, the opportunity for "price discovery."

What about k identical items? What should we do here?

The English Auction for k Identical Items:

Definition 1. In an ascending auction, *sincere bidding* means that a player answers all queries honestly.

Claim 1. In the k identical item setting, in an English auction, sincere bidding is a dominant strategy for every bidder (up to ε).

Claim 2. In the k identical item setting, if all bidders bid sincerely in an English auction, the welfare of the outcome is within $k\varepsilon$ of the maximum possible.

The English auction for k Identical Items terminates in $v_{\max}/\varepsilon$ iterations.

Design process:

1. As a sanity check, design a direct-revelation DSIC welfare-maximizing polytime mechanism.
2. Implement this as an ascending auction.
3. **(Truthfulness)** Check that its EPIC.
4. **(Performance)** Check that it still maximizes welfare under sincere bidding.
5. **(Tractability)** Check that it terminates in a reasonable number of iterations.

Additive Valuations, Parallel Auctions

The Additive Setting: There are m non-identical items and n bidders where each bidder i has private valuation v_{ij} for each item j. Bidder i has an additive valuation for each set S, that is,

$$v_i(S) := \sum_{j \in S} v_{ij}.$$

Step 1: What is the welfare-optimal direct revelation mechanism here?

What's the analogous ascending implementation?

Is this DSIC?

Definition 2. A strategy profile $(\sigma_1, \ldots, \sigma_n)$ is an *ex post Nash equilibrium (EPNE)* if, for every bidder i and valuation $v_i \in V_i$, the strategy $\sigma_i(v_i)$ is a best-response to every strategy profile $\sigma_{-i}(\mathbf{v}_{-i})$ with $\mathbf{v}_{-i} \in \mathbf{V}_{-i}$.

In comparison, in a dominant-strategy equilibrium (DSE), for every bidder i and valuation v_i, the action $\sigma_i(v_i)$ is a best response to every action profile $\mathbf{a}_{-i}$ of $\mathbf{A}_{-i}$, whether of the form $\sigma_{-i}(\mathbf{v}_{-i})$ or not.

Definition 3. A mechanism is *ex post incentive compatible (EPIC)* if sincere bidding is an ex post Nash equilibrium in which all bidders always receive nonnegative utility.

Claim 3. For n additive bidders with m heterogenous items, in parallel English auctions, sincere bidding by all bidders is an ex post Nash equilibrium (up to $m\varepsilon$).

Unit Demand

The Unit-Demand Setting: There are m non-identical items and n bidders where each bidder i has private valuation v_{ij} for each item j. Bidder i is unit demand, that is, wants at most one item for any set S:

$$v_i(S) := \max_{j \in S} v_{ij}.$$

First, solve the direct-revelation problem. What do we observe about the welfare-maximizing allocation in the unit-demand setting?

Refresh yourself on what the VCG mechanism looks like. Then what does the analogous ascending auction look like?