DS 574 Algorithmic Mechanism Design	**Lecture #12**
Fall 2025	Prof. Kira Goldner

Participatory Budgeting

The goal in participatory democracy is to get more people involved in government decisions, especially at the local level. Work-to-date has focused mostly on budgeting decisions, like which capital expenditures to prioritize. For example, residents might be asked whether they'd rather see money spent on improving parks, schools, or public housing. This forces voters to grapple with the types of trade-offs faced by the government. Participatory budgeting is getting increasingly popular—currently, 31 of New York City's 51 local districts use it every year.

k-Approval Voting

The systems currently in place typically use "k-approval voting"—each voter is told the overall budget (e.g., \$1 million) and a list of project descriptions with costs, and the voter picks their k (e.g., 5) favorite projects with no ordering between them.

k-Approval Voting

1. Each voter votes for at most k projects.
2. Sort the projects in decreasing order of number of votes.
3. Fund the maximal prefix such that the total cost is at most the budget B.

What potential issues do we see with this system?

- Voters don't need to take into account the budget. The cost of their k projects can exceed the budget, which over-represents expensive projects.
- There are no preferences expressed among projects.
- Impacted groups are not taken into account.

For example, suppose that the budget is 1 million USD, that $k = 1$, and that all voters have identical values for the three possible projects:

Project Number j	Value $v_{ij}\ \forall i$	Project Cost
1	4	1 million
2	3	500K
3	2	500K

What's the socially optimal outcome? Fund projects 2 and 3.

How might voters vote? What if $k = 2$ instead? Voters will vote for their 1 favorite (project 1). Or top two.

Definition 1. An outcome is *Pareto optimal* if no one can be made better off without making someone else worse off.

Approval voting is not truthful. Approval voting does not necessarily result a Pareto optimal outcome.

Knapsack Voting

We next take a glimpse into the current state-of-the-art, and describe one current prototype for a replacement: knapsack voting [3].[1] The idea is to allow a voter to approve any number of projects, as long as their total cost is at most the budget.

Knapsack Voting

1. Each voter votes for a subset S_i of projects for which the total cost is at most the budget B.[2]
2. Sort the projects in decreasing order of number of votes.
3. Fund the maximal prefix such that the total cost is at most the budget B.

The final project considered—the most popular one that can't be fully funded—will be partially funded, so that the entire budget of B is spent. (This may or may not be realistic.)

Properties of Knapsack Voting

We need to make fairly specific assumptions in order to obtain strategyproofness and Pareto optimality guarantees (otherwise impossibility results kick in).[3] The two assumptions are:

1. Voter i has some set of projects S_i^* that she wants to fund, with the total cost of S_i^* at most the budget B.
2. Voter i wants as much money as possible to be spent on the projects in S_i^*. Thus the utility of voter i is

$$\sum_{j \in S_i^*} [\text{money spent on project } j] \tag{1}$$

[1] See also http://pbstanford.org, and especially the "Boston '16" link to see knapsack voting in action.

[3] There is ongoing work focused on relaxing these assumptions.

The definition (1) implies that if a project (of S_i^*) is partially funded, then the utility earned is pro-rated accordingly.[4] The most unrealistic aspect of this utility model is the extreme assumption that a voter i has absolutely no value for any project outside its preferred set S_i^*.

Under these assumptions, knapsack voting has several nice properties.

Proposition 1. *With voter utilities as in (1), knapsack voting is strategyproof, meaning that a player always maximizes her utility by voting for her true set S_i^*.*

[*Hint:* What happens to projects in S_i^* if this set is misreported?]

Proof Idea. If S_i^* is misreported, some project $j \in S_i^*$ is not reported. Something else may get funded, but it's at the expense of project j, so this lowers i's utility.

Proposition 2. *With voter utilities as in (1), and assuming that voters report their true sets, knapsack voting results in a Pareto optimal choice of projects.*

Proof Idea. In essence, because the algorithm greedily chooses the most voted for alternatives, any other outcome has fewer voters satisfied.

Voting

- Majority-rule: With 2 alternatives, strategyproof.
- Plurality: Most votes wins. This is not strategyproof! Spoiler candidates (e.g., Nader).
- Ranked-Choice Voting (RCV), aka single transferable vote (STV) or instant-runoff voting: There can be an incentive to influence who gets eliminated early on, so that your preferred candidate gets more favored matchups in later rounds.

  On the original preferences, a majority of the voters prefer c to b, and also a majority prefer d to b. So voters might well wonder how b got elected over both c and d. This does not elect the Condorcet winner.
- Borda count: Elects Condorcet winner. But rank your competition last.

  For example, consider the following five votes over the set of alternatives $A = \{a, b, c, d\}$:

	Voters #1,2	Voters #3,4	Voter #5
1st choice	a	b	c
2nd choice	d	a	d
3rd choice	c	d	b
4th choice	b	c	a

Trivial voting rules:

- **Dictator:** a dictator rule has a dictator voter i, and always elects i's first choice.

[4] Also, we assume that no project is ever funded at a level larger than its cost.

- **Duple:** a duple rule chooses a pair $a, b \in A$ of alternatives (independent of voters' preferences), and runs majority vote between a and b.

Theorem 3 (Gibbard-Satterthwaite Theorem [2, 5])**.** *Every strategyproof voting rule that can produce at least three different outcomes is a dictator.*

Theorem 4 (Arrow's Impossibility Theorem [1])**.** *With three or more alternatives, no voting rule satisfies the following three properties:*

- *Non-dictatorship.*
- *Unanimity.*
- *Independent of irrelevant alternatives (IIA).*

Unanimity means that if every voter ranks a over b, then the voting rule should also rank a over b.

IIA means that, for every pair a, b of alternatives, the relative order of a, b in the produced ranking should be a function only of the relative order of a, b in each voter's list, and not depend on the position of any "irrelevant" alternative c in anyone's preferences

Restriction: Single-peaked preferences. Your ranking over alternatives is completely defined by the "distance" from your peak. (E.g., a 1D line on $[0, 1]$.)

A strategyproof voting rule: Report peaks. Take the median of the peaks.

Summary of Other MD4SG Directions

- Housing: low income housing allocation
- Healthcare: kidney exchange, regulating insurance providers, government reimbursing insurance costs (risk adjustment), how to pay doctors
- Economic Inequality: redistribution of wealth
- Online Labor Markets: interview/matching process, information displayed, screen algorithms, bias, wages
- Education: school choice, funding, parental engagement, admissions/hiring, affirmation action
- Sustainability: carbon auctions, prevent pollution, incentivize investment in green technology
- Civic Participation: incentivize voting
- The Developing World: centralizing agriculture markets

Acknowledgements

This lecture was developed in part using materials by Tim Roughgarden, and in particular, his book "Twenty Lectures on Algorithmic Game Theory" [4].

References

[1] Kenneth J Arrow. *Social choice and individual values*, volume 12. Yale university press, 2012.

[2] Allan Gibbard. Manipulation of voting schemes: a general result. *Econometrica: journal of the Econometric Society*, pages 587–601, 1973.

[3] Ashish Goel, Anilesh K Krishnaswamy, Sukolsak Sakshuwong, and Tanja Aitamurto. Knapsack voting for participatory budgeting. *ACM Transactions on Economics and Computation (TEAC)*, 7(2):1–27, 2019.

[4] Tim Roughgarden. *Twenty Lectures on Algorithmic Game Theory*. Cambridge University Press, 2016.

[5] Mark Allen Satterthwaite. Strategy-proofness and arrow's conditions: Existence and correspondence theorems for voting procedures and social welfare functions. *Journal of economic theory*, 10(2):187–217, 1975.