DS 574 Algorithmic Mechanism Design	**Lecture #1 Worksheet**
Fall 2026	Prof. Kira Goldner

Before We Start: Write Down a Number

Exercise.

a. Write down your name and a whole number between 0 and 100, inclusive.

b. The winner is whoever is closest to $\frac{2}{3}$ of the *average* of every number written down in this room.

c. Enter is on the google form via QR code. Don't discuss it with your neighbor.

Eighty Minutes in Gijón

On June 25, 1982, West Germany played Austria in Gijón in the last match of the World Cup group stage. In 1982, during the group stage, there was a round robin (everyone played each other) where wins were worth 2 points, draws worth 1 point, and losses worth 0.

Date	**Match**	**Result**
16 June	Algeria–West Germany	2–1
17 June	Austria–Chile	1–0
20 June	West Germany–Chile	4–1
21 June	Austria–Algeria	2–0
24 June	Algeria–Chile	3–2

Table 1: Group 2 results before the final match.

Team	**P**	**W**	**D**	**L**	**GF**	**GA**	**GD**	**Pts**
Austria	2	2	0	0	3	0	+3	4
Algeria	3	2	0	1	5	5	0	4
West Germany	2	1	0	1	5	3	+2	2
Chile	3	0	0	3	3	8	−5	0

Table 2: Group 2 at kickoff. Algeria and Chile have completed their matches.

Two teams advance edtermined by those with the most points. If teams tie on points, the next differentiator was goal differential (goals scored minus goals let in). Only West Germany and

Austria are left and can do the math, so what should they do?

Result of West Germany–Austria	**Who advances**
West Germany wins by 3 or more	
West Germany wins by 1 or 2	
Draw	
Austria wins	

What should the teams do?

What actually happened on June 25, 1982:

Let's formalize what "utility" means for each team under each result. Write x for the result of the match (for the four rows above). Give each team a reward

$$u_i(x) = \begin{cases} 1 & \text{if} \qquad\qquad\qquad\qquad\qquad \text{under result } x, \\ 0 & \text{otherwise,} \end{cases} \qquad\qquad i \in \{\mathrm{WG}, \mathrm{AUT}\}.$$

What is a best result for West Germany? For Austria?

Exercise. You are FIFA, in July 1982, and this must not happen again. Write down *one* concrete change to the rules.

Your proposals.

Two-Thirds of the Average

Back to your numbers. The average: 2/3 of it: the winner:

What did you actually do to pick your number?

Why are people choosing different strategies? What's happening here?

The model. Each of the n people in the room picks a number $b_i \in [0, 100]$. Call b_i player i's *strategy*, and call the vector $b = (b_1, \ldots, b_n)$ of everyone's choices a *strategy profile.* The target is $t(b) = \frac{2}{3} \cdot \frac{1}{n} \sum_{j=1}^{n} b_j$. Player i's *payoff* is $-|b_i - t(b)|$: closer is better. Note: your own number is part of the average.

Definition 1. Strategy b_i is *strictly dominated* by strategy b_i' if, no matter what everybody else does, i's payoff from b_i' is strictly larger than i's payoff from b_i.

That is, a strictly dominated strategy is one you can throw away without knowing the other players.

Theorem 1. *Suppose every player's number lies in* $[0, M]$ *for some* $M > 0$*. Then for player* i*, every strategy* $b_i > \frac{2}{3}M$ *is strictly dominated by the strategy* $\frac{2}{3}M$*.*

Proof.

Corollary 2. *Applying Theorem 1 with* $M_0 = 100$*, no rational player picks above* $M_1 = \frac{2}{3} \cdot 100 = 66.\overline{6}$*. If nobody picks above* M_1*, the theorem applies again with* $M = M_1$ *and nobody picks above* $M_2 = \frac{2}{3}M_1 = 44.4$*; then* $M_3 = 29.6$*,* $M_4 = 19.8$*, and in general* $M_k = 100\left(\frac{2}{3}\right)^k \to 0$*. The only number surviving every round of this elimination is* 0*.*

Results.

Thought experiment: suppose we were to play again, right now. What would happen to the average?

Takeaway: A mechanism has to work on .

Split or Steal

Golden Balls ran on ITV from 2007 to 2009. In the last sixty seconds of an episode, two contestants have built a jackpot of J pounds. Each secretly picks a ball marked *Split* or *Steal*. Then if: both Split, and they split the jackpot, $J/2$ each; one Splits and one Steals, and the Stealer takes all of J while the Splitter gets nothing; both Steal, and both get nothing.

Exercise. In pairs, competing for a jackpot, you may talk to each other for ninety seconds, and then you choose simultaneously.

Before the exercise—your prediction, and why:

Now let's write it down. Rows are your choice, columns are theirs, and each cell is (your payoff, their payoff):

	They Split	**They Steal**
You Split	$(J/2,\ J/2)$	$(0,\ J)$
You Steal	$(J,\ 0)$	$(0,\ 0)$

Definition 2 (Weakly Dominated)**.**

Claim 1. In *Golden Balls* with jackpot $J > 0$, Split is weakly dominated by Steal. It is not strictly dominated.

Proof.

Why do we care about the difference between weak and strict here?

The Nick Corrigan / Ibrahim Hussein episode:

Work out Ibrahim's problem. Suppose Ibrahim believes Nick. Let $q \in [0, 1]$ be the chance that Nick actually hands over half afterward.Then Ibrahim's payoffs are

Split: , Steal: ,

and so Split is strictly better if and only if .

What's happening here?

What We Observed Today

Definition 3. A *mechanism* is a rule x that maps the players' actions $b = (b_1, \ldots, b_n)$ to an outcome $x(b)$, together with the rewards each player receives from that outcome.

Mechanisms include: tournament formats, points formulas, auctions, school-assignment systems, ballots, promotion policies, recommendation algorithms, etc. Designing one means choosing x knowing that .

That is the whole difference between designing a mechanism and designing an algorithm: .

So the primary question in this course is: *when can we design a rule so that the behavior we want is the behavior the players choose?*

And another observation of today is: